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MULTIPRODUCT COST-VOLUME-PROFIT ANALYSIS: MATHEMATICAL REPRESENTATION OF CLASSICAL LINEAR MODELS

ABSTRACT

The subject of research in the paper is classical multiproduct Cost-Volume-Profit (MCVP) analysis models. Methods of mathematical representation and proof, as well as empirical verification on a selected specific case from practice, were used. It has been mathematically shown that the known linear relational MCVP model, which starts from the equation of operating income, and the model that considers the average unit margin, results in identical solutions that can be substituted with one general solution. This way, the determined general solution can be used also for the financial calculation of the multiproduct break-even point. Regarding the use of contribution coefficients of individual products, the derived general MCVP models can cover absolute or relative relations in such a way that a defined structure (historical or target) of the production volume relations of individual products is maintained, that no product is declared as the base or that any product is declared as the base product. Also, the obtained general MCVP models provide solutions in one step, so they are simpler to understand and implement compared to classical approaches, which can improve the efficiency of their use.

Keywords: *multiproduct cost-volume-profit analysis, break-even point, linear model*

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1. INTRODUCTION

Technological innovations, global competition, and changes in the environment pose demands on management for the permanent availability of relevant information, both in the short and long term, essential for managing business results. One of the oldest analytical techniques for managing profitability is the analysis of the break-even point or more commonly CVP (Cost-Volume-Profit) analysis (Wilks, Burke, 2005). In business reality, enterprises that produce/sell more than one product dominate, which increases the complexity of the CVP analysis (Hilton, 2008). The multiproduct situation is characterized by a combination of the production of related or different products that the enterprise produces over a certain period (Atkinson, Kaplan, 2007; Gulin et al., 2011; Stevanović, Petrović, 2016).

The significance of CVP analysis as an analytical instrument is confirmed by its presence in the contents of almost all contemporary textbooks in the fields of managerial accounting, controlling, microeconomics, marketing metrics, etc. Thus, in textbooks on managerial accounting (e.g., Horngren et al., 2002; Atkinson, Kaplan, 2007; Wilks, Burke, Hilton, 2008; Gulin et al., 2011; Malinić, Milićević, Stevanović, 2013; Bhimani et al., 2018), various aspects of the break-even point of one product are analyzed in detail. However, multiproduct CVP (MCVP) analysis is very modestly covered although it is more immanent to real business situations. Generally, two linear MCVP models are mentioned; namely, the model that starts from the equation of operating income and the model that is based on a weighted-average contribution margin. These models can be called classical linear MCVP models. Unlike these approaches, we believe that there is a lack of mathematical analysis and generalization of these classical linear MCVP models, as well as a comparative analysis of the obtained general solutions. Taking into account the above, the main objectives of the paper are the following:

- Based on classical procedures for determining the multiproduct break-even point, derive general quantitative models and mathematically demonstrate the identity of the final formulas and obtained results.
- Generalize the known procedure for calculating the value-expressed multiproduct break-even point and mathematically confirm the identity of the obtained formulas and results.
- Empirically test the obtained results by analyzing a case from practice.

Although the identified gaps seem clear and simple, their resolution requires the use of mathematical representation and proof methods. Therefore, this paper aims to contribute to the understanding of classical linear MCVP analysis and to simplify the analysis process itself.

In addition to the introduction and conclusion, the paper is organized into three parts. The first part contains a review of the literature. The second part presents a mathematical analysis of classical linear multiproduct break-even point models, which resulted in deriving general formulas for MCVP analysis. The third part relates to the empirical analysis of a real multiproduct situation and a discussion of the research results.

2. LITERATURE REVIEW

The basic settings of single-product linear CVP analysis were presented at the beginning of the 20th century by Hess and Maan (Stefan, 2012). Since then, CVP analysis as a business performance assessment tool has progressively been theoretically and methodologically enhanced, leading to the development of various approaches and models. Linear deterministic models hold a significant place in MCVP analysis approaches (Gonzalez, 2001; Kucharski, Wywiał, 2019; Wijayanti, Prasetyo, 2021), whose applicability in the short term is emphasized by numerous authors (Atiase, 1989; Gonzales, 2001; Drury, 2011).

Considering the characteristics of the production assortment, CVP models can be differentiated into homogeneous and non-homogeneous production. In situations where a company produces a larger number of different products that can be expressed in the same physical unit of measure (homogeneous assortment), a single-product CVP model can be applied (Malinić, Milićević, Stevanović, 2013). Non-homogeneous (heterogeneous) production is considered in the context of a production assortment with several different products (Kucharski, Wywiał, 2019; Potkany, Krajcirova, 2015), so that CVP analysis of non-homogeneous production and MCVP analysis can be treated as synonyms.

Respecting the objectives of resolution, relational and optimization MCVP models are distinguished. Relational or non-optimization linear MCVP models contain relational equations between model parameters to calculate the multiproduct break-even point, without establishing optimization criteria. The most common linear relational models are the model that starts from the operating income equation and the model of the weighted-average contribution margin, and their various variants (Wilks, Burke, 2005, Hilton, 2008, Malinić, Milićević, Stevanović, 2013; Kim, 2015; Potkany, Krajcirova, 2015; Stevanović, Petrović, 2016). In the model starting from the operating income equation, using an equivalent conversion factor, the calculated multiproduct break-even point provides a unique solution if any product is chosen as the base product (Ngamsomsuke, Ngamsomsuke, Rabten, 2022). On the other hand, optimization models are used to determine the profitability point for multiple products with the aim of finding the optimal combination of the product assortment where an extreme value (maximum or minimum) of the objective function (criteria) is achieved. Optimality criteria in these models can include maximizing total revenue or profit (Briciu, Căpușneanu, Căprariu, 2013; Kucharski, Wywiał, 2019), minimizing production costs (Zhang, Yang, Chai, Qiu, 2012; Kucharski, Wywiał, 2019), etc. Linear optimization models are most commonly solved using linear programming methods (Kucharski, Wywiał, 2019; Wijayanti, Prasetyo, 2021). In the article (Zahirović, Okičić, Gadžo, 2024) it was shown empirically that relational models and linear programming models in a multiproduct situation give an identical solution assuming the constancy of the structure of the physical volume of production, total revenue, total costs and total margins.

Considering the time factor, linear MCVP analysis can pertain to a single relevant period (static models) or cover a time course by encompassing several relevant periods (dynamic models). Static MCVP analysis contains the majority of the aforementioned linear and optimization models. Dynamic models, on the other hand, include variable time factors, allowing for analysis of how the break-even point changes over time with changes in costs, prices, demand, capacity utilization, etc. Guang-bin, Bin-li (2007) believe that the time value of money should be included in MCVP analysis, and on these bases, they formulate a mathematical model. Additionally, there are other studies where models for dynamic MCVP analysis have been developed and their applicability examined (Prihadyanti, 2011).

In terms of cost accounting methods, linear MCVP analysis can be based on traditional and contemporary costing systems. All previously highlighted linear models are based on the traditional costing system. However, an alternative MCVP approach that includes modern costing systems has also been developed, among which the most significant role is played by activity-based costing (González, 2001; Kee, 2001; Wen-Hsien et al., 2013).

This literature review points to a multitude of different linear approaches that can cope with a multiproduct situation. On the other hand, predominantly based on the criticism of linear models, and considering various assumptions, more complex MCVP approaches such as nonlinear (Ndaliman, Bala, 2007), stochastic (Asih, Eng, 2021), based on fuzzy logic (Lazzari, Moriñigo, 2003; Fong-Ching, 2009; Baral, 2016; Aslan, Yilmaz, 2018) and integral MCVP models (Horal, Shyiko, Yaroshenko, 2019) have progressively been developed. These models are often combined with various optimization techniques.

3. MATHEMATICAL REPRESENTATION OF CLASSICAL LINEAR MODELS OF THE MULTIPRODUCT BREAK-EVEN POINT

The basic mathematical assumptions of linear models in MCVP analysis are: a priori known production/sales mix, fixed costs are constant, variable costs and revenues from individual products change proportionally to the change in the physical volume of production, i.e., they behave linearly (Horngren, Foster, Datar, 2002; Gulin et al., 2011; Malinić, Milićević, Stevanović, 2013; Bhimani et al., 2018).

If we denote with:

q_i –volume of production of the i-th product at the break-even point for the relevant period

(relevant range of activity),

p_i – the unit selling price of the i-th product,

w_i – the unit variable cost of the i-th product,

F – total fixed costs in the relevant period,

$m_i = p_i - w_i$ – the unit margin achieved by producing/selling the i-th product,

$i = \overline{1,n}$,

the key equation used to determine the multiproduct break-even point with n products can be written in the form:

$\sum_{i=1}^{n}(p_i - w_i)q_i = F$ or (1)

$\sum_{i=1}^{n} m_i q_i = F.$ (2)

Taking into account the non-negative values of the variable $q_i \geq 0,\ i = \overline{1,n}$, equation (1) in the *n*-dimensional vector space, represents a hyperpolygon that has the properties of a convex set, and represents the set of possible solutions (Zahirović S., Kozarević S., 2022). Every combination of production volumes of individual products q_i where equation (1) or (2) is satisfied represents a multiproduct break-even point, so the number of possible multiproduct combinations is theoretically infinite. Therefore, to develop concrete models of the multiproduct break-even point, it is necessary to introduce additional mathematical constraints or assumptions.

For classical linear MCVP models, the additional mathematical assumption relates to the constancy of the ratio of the physical volume of production of individual products in the following form:

$q_1 : q_2 : \ldots : q_n = k_1^v : k_2^v : \ldots : k_n^v,$ (3)

where is k_i^v – the coefficient of proportionality for the production volume of the *i*-th product.

3.1. MCVP MODEL BASED ON THE OPERATING INCOME EQUATION

In the literature (Horngren, Foster, Datar, 2002; Malinić, Milićević, Stevanović, 2013; Bhimani et al., 2018; Gulin et al., 2011), procedures for calculating the multiproduct break-even point starting from the operating income equation (1) and assuming constant ratios of the physical volume of production of individual products (4) are provided. Such a model can be called an MCVP model based on the basic (conditional or standard) product.

(a) If q denotes the production volume of the basic (conditional or standard) product, then the equivalent conversion of the production volume of the i-th product can be expressed in the form of the relation:

$$q_i = k_i^v q,\ i = \overline{1,n}. \tag{4}$$

By substituting expression (4) into equation (1), the equation obtained is:

$$\sum_{j=1}^{n}(p_j - w_j)k_j^v q = F, \tag{5}$$

based on which the expression for calculating the quantity of the conditional product is obtained in the form:

$$q = \frac{F}{\sum_{j=1}^{n} k_j^v (p_j - w_j)}. \tag{6}$$

Based on the calculated quantity of the conditional product (6), and using expression (4) for $i = \overline{1,n}$ the break-even point calculation of the production volume of individual products can be performed q_i in a multiproduct situation.

(b) Alternatively, starting from the business profit equation, the formula for calculating the production volume i-th of products in the production mix that ensures reaching the multiproduct break-even point is obtained by substituting expression (6) into (4) in the following form (Zahirović, Okičić, Gadžo, 2024):

$$q_i = \frac{k_i^v F}{\sum_{j=1}^{n} k_j^v (p_j - w_j)},\ i = \overline{1,n}. \tag{7}$$

This solution is identical to the solution obtained based on expressions (6) and (4), so we can call it the general solution for the multiproduct break-even point with constant ratios.

3.2. MULTIPRODUCT BREAK-EVEN MODEL BASED ON THE WEIGHTED-AVERAGE CONTRIBUTION MARGIN

The procedure for calculating the multiproduct break-even point based on the weighted-average contribution margin is given in the works of Horngren, Foster, Datar (2002), Hilton (2008), Malinić, Milićević, Stevanović, (2013), Enyi (2019), and Babiak, Krutous (2021).

(a) If $\overline{m}$ denotes the weighted-average unit contribution margin, then the expression for its calculation would be as follows:

$$\overline{m} = \frac{\sum_{j=1}^{n} k_j^v (p_j - w_j)}{\sum_{j=1}^{n} k_j^v}. \tag{8}$$

The total quantity (sum) of the production volume of individual products is obtained using the expression:

$$\sum_{j=1}^{n} q_j = \frac{F}{\bar{m}}. \tag{9}$$

Taking into account the ratio of the proportionality coefficient k_i^v of the production volume of the i-th product and the sum of all coefficients from expression (3), and based on expression (9), the formula for calculating the production volume of individual products in a multiproduct situation is obtained in the form:

$$q_i = \frac{k_i^v}{\sum_{j=1}^{n} k_j^v} \sum_{j=1}^{n} q_j, \; i = \overline{1,n}. \tag{10}$$

If we substitute expression (8) into expression (9), and then substitute that result into expression (10), we obtain the general formula which is identical to expression (7).

(b) An alternative procedure for obtaining the general formula for calculating the multiproduct break-even point, taking into account the weighted-average unit contribution margin, starts from the expression valid for proportion (3):

$$q_i = Const \cdot k_i^v, \; i = \overline{1,n}, \tag{11}$$

where $Const$ denotes the arbitrarily chosen constant.

Based on expression (11), using the summation operator, we get:

$$\sum_{j=1}^{n} k_j^v = \frac{1}{Const} \sum_{j=1}^{n} q_j. \tag{12}$$

By substituting expression (11) into expression (1), the following equation is obtained:

$$Const \sum_{j=1}^{n} k_j^v (p_j - w_j) = F, \tag{13}$$

which, by expanding the expression on the left side, can be written in the form:

$$Const \sum_{j=1}^{n} k_j^v \frac{\sum_{j=1}^{n} k_j^v (p_j - w_j)}{\sum_{j=1}^{n} k_j^v} = F. \tag{14}$$

By substituting expression (12) into expression (14) and simplifying it, the equation obtained is:

$$\sum_{j=1}^{n} q_j \frac{\sum_{j=1}^{n} k_j^v (p_j - w_j)}{\sum_{j=1}^{n} k_j^v} = F. \tag{15}$$

Given that the multiplication factor on the left side of equation (15) represents the weighted-average unit contribution margin $\bar{m}$ as given by expression (8), equation (15) can be written in the form:

$$\bar{m} \sum_{j=1}^{n} q_j = F, \tag{16}$$

which is identical to expression (9).

By solving equation (15) $\sum_{j=1}^{n} q_j$ the following expression is obtained:

$$\sum_{j=1}^{n} q_j = \frac{F\sum_{j=1}^{n} k_j^v}{\sum_{j=1}^{n} k_j^v (p_j - w_j)}. \tag{17}$$

Based on the property of the proportion (3) that $q_i : k_i^v = \sum_{j=1}^{n} q_j : \sum_{j=1}^{n} k_j^v,\ i = \overline{1,n}$, the following expression is obtained:

$$q_i = \frac{k_i^v}{\sum_{j=1}^{n} k_j^v} \sum_{j=1}^{n} q_j, \tag{18}$$

which is identical to expression (10). By substituting expression (17) into expression (18), the mathematical formula for calculating the multiproduct break-even point for the production volume of the *i*-th product in the general form, which is identical to expression (7), is obtained.

Taking into account the different approaches presented in sections 3.1 and 3.2, it is evident that all used approaches for obtaining the multiproduct break-even point for the production volume of the i-th product lead to the identical solution given by relation (7). Therefore, this formula is rightly called the general solution for the multiproduct break-even point with constant ratios.

3.3. VALUE-EXPRESSED MULTIPRODUCT BREAK-EVEN MODEL

(a) Since we have already obtained the mathematical formula for calculating the multiproduct break-even point for the production volume of the i-th product (7), the value-expressed break-even point for the revenue from the sale of the i-th product $p_i q_i$ can be obtained simply by multiplying this solution by p_i. Thus, the final formula for the partial revenueof the i-th product (r_i) is obtained in the following form:

$$r_i = p_i q_i = \frac{k_i^v p_i F}{\sum_{j=1}^{n} k_j^v (p_j - w_j)}. \tag{19}$$

(b) An alternative procedure for calculating the value-expressed multiproduct break-even point is given in the literature (Horngren, Foster, Datar, 2002; Malinić, Milićević, Stevanović, 2013). Using mathematical formulas, this procedure begins with calculating the weighted-average contribution margin ratio $\overline{m}_s$ using the relation:

$$\overline{m}_s = \frac{\sum_{j=1}^{n} k_j^v (p_j - w_j)}{\sum_{j=1}^{n} k_j^v p_j}. \tag{20}$$

The total revenue or the value-expressed break-even point for all n products is calculated based on the expression:

$$\sum_{j=1}^{n} p_j q_j = \frac{F}{\overline{m}_s}. \tag{21}$$

To determine the value-expressed multiproduct break-even point using the method of the weighted-average contribution margin ratio, it is necessary to introduce an additional assumption about the constancy of the revenue ratios of individual products in the production mix, i.e.,

$$p_1q_1 : p_2q_2 : \ldots : p_nq_n = k_1^r : k_2^r : \ldots : k_n^r, \quad (22)$$

where k_i^r – is the proportionality coefficient of the partial revenue of the i-th product.

Finally, the value of the partial revenue of the i-th product at the multiproduct break-even point is calculated based on the product of the participation of the proportionality coefficient of the revenue of the i-th product in the sum of the revenue coefficients of all n products and the total revenue for all n products (21), namely:

$$r_i = \frac{k_i^r}{\sum_{j=1}^{n} k_j^r} \sum_{j=1}^{n} p_j q_j. \quad (23)$$

To mathematically confirm the identity of the solution for the revenue value i-th of products given by relations (19) and (23), it is necessary to establish a connection between the proportionality coefficients k_i^v i k_i^r.

Based on the relations $q_i = Const1 \cdot k_i^v$ i $p_i q_i = Const2 \cdot k_i^r$, $i = \overline{1,n}$, the following expression is obtained:

$$k_i^r = \frac{Const1}{Const2} k_i^v p_i, \quad (24)$$

where $Const1$ and $Const2$ denote the arbitrarily chosen constants.

The sum of the coefficients k_i^r can be expressed as:

$$\sum_{j=1}^{n} k_j^r = \frac{Const1}{Const2} \sum_{j=1}^{n} k_j^v p_j. \quad (25)$$

By substituting expressions (24) and (25) into expression (23), the formula for the value of the partial revenue of the i-th product at the multiproduct break-even point, which is identical to expression (19), is obtained, as needed to be shown. Therefore, the solution (19) can be called the general solution for the value-expressed multiproduct break-even point.

4. CASE STUDY ANALYSIS OF MULTIPRODUCT BREAK-EVEN CALCULATION

Although the mathematical analysis has shown that all the presented models lead to an identical solution, we considered it purposeful to empirically interpret all the presented results as well.

4.1. DATA

For the purpose of empirical testing, a company from the metal processing industry was selected. The first reason is the management's willingness to participate in the study and to provide the necessary data, some of which are business secrets. The second reason relates to the stability of the company's operating conditions, which is a prerequisite for the adequate application of linear models.

The selected company produces 5 products Pi, $i = \overline{1, 5}$, which are predominantly sold on the international market. The total fixed costs (F) for the relevant annual period amounted to 11.321.340 Euros. Data were collected on the production volumes of individual products achieved in the previous year, unit selling prices, and unit variable costs of individual products.

4.2. RESULTS AND INTERPRETATION

In Table 1, the following are presented:

- Data on the production volumes of individual products achieved in the previous year (Qi), unit selling prices (p_i) and unit variable costs (w_i) of products Pi.
- Calculated proportionality coefficients of the physical production volume (k_i^v) and revenues of individual products (k_i^r), assuming the maintenance of constant ratios of physical production volume (the base product is P5) and revenues (the base product is P3) of individual products in the production mix.
- Calculated other necessary values for the calculation of the multiproduct break-even point.
- Results of calculating the multiproduct break-even point for individual products expressed in physical units of measure (q_i) and value (r_i).

Table 1. *Model testing - Multiproduct break-even point analysis*

Description	Product type					Σ
	P_1	P_2	P_3	P_4	P_5	
Q_i	3.242	2.100	250	832	56	
p_i	4 328	4 625	2 980	5 894	15 242	
w_i	1 950	2 140	2 430	3 534	12 150	
m_i	2 378	2 485	550	2 360	3 092	
k_i^v	57,893	37,5	4,464	14,857	1	115,714
$k_i^v m_i$	137 669,55	93 187,5	2 455,2	35 062,52	3 092	271 466,77
$k_i^v p_i$	250 560,90	173 437,5	13 302,72	87 567,16	15 242	540 110,28
$p_i Q_i$	14 031 376	9 712 500	745 000	4 903 808	853 552	30 246 236
k_i^r	18,8340617	13,0369128	1	6,58229262	1,14570738	40,5989745
q_i	2 414	1 564	186	620	42	4 826
r_i	10 449 474,7	7 233 094,8	554 781,03	3 651 929,68	635 657,40	22 524 937,6

Note: The calculation results have been rounded.

Source: *Author's processing*

Based on the MCVP model that starts from the operating income equation defined by relation (5), the following equation is obtained where the operating income is equal to zero:

137 669,55q+93 187,5q+2 455,2q+35 062,52q+3 092q=11 321 340.

By solving the previous equation, the solution for the basic product is obtained (q=41,70). By substituting this value into expression (4), the production volume of products at the multiproduct break-even point is calculated (row q_i). Identical results are obtained by applying expression (7).

Using the model that starts from the weighted-average unit contribution margin first, based on expression (8), $\overline{m} = 2\ 346{,}01$ is calculated. The total sum of the production volumes of individual products is obtained using expression (9) ($\sum_{i=1}^{5} q_i = 4\ 826$). Based on expression (10), the production volume of individual products in a multiproduct situation can be calculated (row q_i).

The partial revenue of products Pi , $i = \overline{1,5}$ at the multiproduct break-even point in one step, can be calculated using formula (19) (row r_i). The alternative procedure starts by calculating the weighted-average contribution margin ratio $\overline{m}_s$ for all five products using expression (20) ($\overline{m}_s = 0{,}502613601$), then based on expression (21) calculates the total revenue for all five products ($\sum_{i=1}^{5} p_i q_i = 22\ 524\ 937{,}6\ Euros$), and using expression (23) obtains the partial revenue of individual products at the multiproduct break-even point.

4.3. DISCUSSION AND RESEARCH LIMITATIONS

Although multiproduct CVP analysis based on deterministic and linear models has been known for a long time, the results obtained in this study show that these models can still be explored and improved. The paper mathematically proved that the well-known linear MCVP model starting from the business profit equation and the model that takes into account the weighted-average contribution margin in an identical solution given by equation (7). Therefore, this solution can be called a general (universal) solution or a general relational model for the multiproduct break-even point with constant ratios of the physical production volume of individual products. It was also shown that the general solution (7), calculated in physical units, can also be used for the financial calculation of the break-even point using the general relation (19). Regarding the use of the contribution ratio coefficients of individual products, the general models (7) and (19) can apply to any real or target (planned) ratio of partial physical production volumes, i.e., they do not necessarily require a reduction to a basic or conditional product, which is more comprehensive compared to the results given in the works of Enyi (2019), Babiak, Krutous (2021), and Ngamsomsuke, Ngamsomsuke, Rabten (2022). This means that MCVP analysis based on the developed general models can be conducted in such a way that the defined structure (historical or target) of the participation of individual products expressed in physical units of measure is maintained, that no product is declared as basic, or that any product is declared as the basic product (relative ratios). Furthermore, unlike the MCVP methodology given in the works (Horngren, Foster, Datar, 2002; Wilks, Burke, 2005; Hilton, 2008; Malinić, Milićević, Stevanović, 2013), in which the calculation of the multiproduct break-even point is carried out in several steps, the developed general models provide solutions in one step. The obtained solutions (7) and (19) generalized the methodology of multiproduct CVP linear analysis with constant contribution structures of individual products, thereby expanding the theoretical and methodological knowledge of deterministic and linear relational models (Horngren, Foster, Datar, 2002; Hilton, 2008; Gulin et al., 2011; Malinić, Milićević, Stevanović, 2013).

Generally speaking, CVP analysis for heterogeneous production is more complex compared to homogeneous production, which also applies to linear models that assume constant ratios of physical production volume. Compared to nonlinear, dynamic, and stochastic CVP models, the developed general linear models are simpler to understand and implement, which is consistent with the findings presented in the works of Babiak, Krutous (2021). They provide opportunities for relatively simple and direct MCVP analysis, requiring knowledge of unit selling prices, unit variable costs, total fixed costs, and contribution coefficients of individual products. The conducted empirical analysis on a specific case confirmed the possibilities for simple, efficient, direct, and accurate calculation of the multiproduct break-even point (Kim, 2015; Potkany, Krajcirova, 2015; Babiak, Krutous, 2021). Sensitivity analysis and simulation of the potential contribution coefficients of individual products can yield solutions for different scenarios, which can improve the scope and accuracy of results regarding a wider range of possible outcomes in a relatively predictable environment. The developed generalized methodology of MCVP analysis allows for the expansion of the methodological toolkit for production and financial planning and decision support.

The research results should be interpreted in the context of limitations. The most significant limitation is related to the assumptions of linear and deterministic relational models regarding the constancy of the realizations of basic input variables and their relationships, thus not covering risk and uncertainty, i.e., these approaches do not contain nonlinear, variable, dynamic, or stochastic characteristics of multiproduct systems often present in a real business environment. Furthermore, the developed MCVP models assumed traditional cost accounting, so more precise results could be obtained if some modern approaches, such as activity-based costing analysis, were applied.

5. CONCLUSION

In this paper, an MCVP approach was systematized, and a classification of linear MCVP models was proposed based on the following criteria: the size of the product assortment, the objectives of solving the model, the inclusion of the time variable, and the method of cost accounting. Known classical linear relational models of the multiproduct break-even point with constant contribution ratios of individual products were mathematically and empirically analyzed. The results of the mathematical representation of the model based on the operating income equation and the model that starts from the weighted-average contribution margin can be substituted using only one general equation. Therefore, the key theoretical-methodological contribution of the work consists of expanding the analytical tools that enhance the understanding of the multiproduct break-even point, thereby enriching the existing literature in terms of the generalization of MCVP models and the development of business analytics and financial metrics. In this way, the practical applicability of MCVP analysis can be simplified and made more efficient. The implementation of the developed general linear MCVP models has the potential for a quick evaluation of the current state of the company in terms of the production/sales mix and profitability management, as well as business strategy planning, defining sales targets, and so on. Of course, when planning and making decisions based on linear MCVP analysis, it is always necessary to keep in mind the lack of flexibility of the models to cope with uncertainty and variability of market conditions, which can lead to inaccurate predictions in a dynamic and uncertain environment.

Future research could focus on developing new linear models for point and interval estimation of the multiproduct break-even point that would encompass more realistic assumptions about the behavior and relationships of inputs in the analysis, as well as the specificities of the market and operational environment of different industrial sectors. Additionally, research could focus on the theoretical and methodological aspects of optimization, nonlinear, dynamic, and stochastic MCVP models that can handle the complexity of multiproduct systems. This would include the possible development of integral MCVP models incorporating characteristics of different approaches where, due to their efficiency and simplicity, the initial multiproduct solution could be calculated based on the developed MCVP models, and other solutions could be iteratively improved and expanded, while taking into account other assumptions.

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VIŠEPROIZVODNA ANALIZA TROŠKA-VOLUMENA-PROFITA: MATEMATIČKA REPREZENTACIJA KLASIČNIH LINEARNIH MODELA

SAŽETAK

Predmet istraživanja u radu su klasični modeli višeproizvodne analize troška-volumena-profita (MCVP). Korištene su metode matematičke reprezentacije i dokazivanja, te emprijske verifikacije na odabranom konkretnom slučaju iz prakse. Matematički je pokazano da poznati linearni relacioni MCVP model koji polaze od jednačine poslovnog dobitka i model koji uzima u obzir prosječnu jediničnu marginu rezultiraju identičnim rješenjima koja se mogu supstituirati sa jednim opštim rješenjem. Na ovaj način određeno opšte rješenje može se koristiti i za finansijski obračun višeproizvodne tačke pokrića. U pogledu korištenja koeficijenata doprinosa pojedinačnih proizvoda, izvedeni opšti MCVP modeli mogu obuhvatiti apsolutne ili relativne odnose na način da se zadrži definisana (historijska ili ciljna) struktura odnosa obima proizvodnje pojedinačnih proizvoda, da se nijedan proizvod ne proglasi kao osnovni ili da se bilo koji proizvod proglasi osnovnim proizvodom. Također, dobiveni opšti MCVP modeli daju rješenja u jednom koraku, tako da su u odnosu na klasične pristupe jednostavniji za razumijevanje i implementaciju, što može unaprijediti efikasnost njihovog korištenja.

Ključne riječi: *višeproizvodna analiza troška-volumena-profita, tačaka pokrića, linearni model*

JEL: *M40, C61*